Statement of Kashiwara-Vergne

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$$x + y - \log e^y e^x = (1 - e^{-\operatorname{ad} x})F + (e^{\operatorname{ad} y} - 1)G$$
$$\operatorname{tr}(\operatorname{ad} x)\partial_x F + \operatorname{tr}(\operatorname{ad} y)\partial_y G = \frac{1}{2}\operatorname{tr}\left(\frac{\operatorname{ad} x}{e^{\operatorname{ad} x}-1} + \frac{\operatorname{ad} y}{e^{\operatorname{ad} y}-1} - \frac{\operatorname{ad} z}{e^{\operatorname{ad} z}-1} - 1\right)$$
$$\text{with } z = \log e^x e^y$$

Torossian's version: $(Z(x,y) := \log e^x e^y)$

$$\int u(x)v(y)\frac{j^{1/2}(x)j^{1/2}(y)}{j^{1/2}(Z(x,y))}f(Z(x,y))dxdy = \int u(x)v(y)f(x+y)dxdy$$

convolution: The measure $\mu * \nu$ is defined by

$$\int f\, \mu * \nu = \int f(x+y)\mu(x)\mu(y)$$

Take $f := j^{1/2} g$ get

$$\int u(x)v(y)j^{1/2}(x)j^{1/2}(y)g(\log e^x e^y) = \int u(x)v(y)j^{1/2}(x+y)g(x+y)$$

take $g(\log x) = h(x)$ get $[g(z) = h(e^z)]$

$$\int u(x)v(y)j^{1/2}(x)j^{1/2}(y)h(e^x e^y) = \int u(x)v(y)j^{1/2}(x+y)h(e^{x+y})$$

This is my version, with $w^2 = j^{1/2}$

(as in the Glasgow handout)
or Trieste
or Paris